DOCUMENT RESUME

ED 068 315 SE 014 717

AUTHOR Barczyk, Gloria J.
TITLE A Teacher's Approach to Adjusting Instruction in Elementary School Mathematics to Varied Ability Groups.
PUB DATE [71]
NOTE 78p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Behavioral Objectives; *Curriculum; Curriculum Guides; Elementary Grades; *Elementary School Mathematics; Grade 5; *Individualized Instruction; Instruction; *Instructional Materials; Lesson Plans; Mathematics Education; *Teacher Developed Materials; Worksheets
IDENTIFIERS Systems Approach to Mathematics Instruction (SAM)

ABSTRACT

This paper is concerned with developing a mathematics curriculum for the fifth grade which uses a program of varied difficulty of instruction based on "A Systems Approach to Improving Mathematics Instruction" (SAM), a program developed in the Pittsburgh area. The first portion of the paper is a general discussion of facets involved in curriculum construction. The remainder of the paper details the specific objectives, the selecting and sequencing of content, and the instructional organization of a fifth grade mathematics course. Sample materials are included: a "Curriculum Suggested Pace" which lists the basic levels of instruction as well as suggested enrichment topics for each level; behavioral objectives for each basic level; and the complete lesson plans along with teacher-constructed materials for two of the topics covered in the curriculum (fractions and negative numbers). (DT)

ED 068315

A Teacher's Approach to Adjusting Instruction in Elementary School Mathematics to Varied Ability Groups

Gloria J. Barczyk

Resurrection School
Pittsburgh Parochial Schools
Pittsburgh, Pennsylvania

INTRODUCTION

A problem of continuing concern in the teaching profession is that of adjusting instruction to meet the individual needs of each pupil. Humanistic education, child-centered curriculum, and individualized instruction are all terms of high significance and utility today. These terms suggest a trend to make education individually unique. Yet this idea is not a new one to the history of education. It has long been a reality that pupils differ from one another in terms of abilities and interests, but few schools have accepted the responsibility for meeting these individual differences. Part of the reason that underlies this misfortune is that for all practical purposes, the most efficient way of dealing with the education of a large number of children (which most certainly was the case with the initiation of compulsory education) has been to meet the needs of the group rather than the needs of the individual. Educators have come to realize, however, that mass group instruction does not necessarily meet the needs of pupils with exceptionally high ability, or the needs of the very slow learner. These humanitarians are challenging the morality of an educational system which does not, in practice, strive for the fullest development of each individual.

In addition to organizational techniques of grouping for instruction, some educators are also questioning the relevancy of certain content material in the subject areas. Their concepts of what can and should be taught to children are changing radically (Frost & Rowland, p.334).

Elementary school mathematics has gone through several such changes which have influenced textbook writers and others responsible for the mathematics curriculum. The usefulness and grade placement of certain topics in elementary school arithmetic has been questioned by educators as the concern for the child as an individual, and as a learner, has increased. As a result of research done by psychologists such as Piaget, and the experience of teachers in classrooms, the introduction of certain topics has been adjusted to develop better learning situations. Likewise, topics in arithmetic believed to have a low utility value in daily life, have been omitted and an effort has been made to identify the learning rates of students that correspond with their abilities (Kramer, p.4).

Research in educational psychology has also suggested the need for reform in teaching procedures. As a reaction against a method based largely on repetition, some mathematics programs experimented with incidental teaching, where the child's need was the chief reason for learning. It was soon concluded, however, that mathematics could not be learned well by either drill or the incidental approach alone, nor even by the two combined (Kramer, p.5). It was suggested that if mathematics is to provide a system for searching out patterns which can be used to arrive at solutions to problems, it must include a powerful method which rests on abstractions and generalizations of a higher order than that of rote memorization of simple arithmetic algorithms. For this reason, modern mathematics programs have come to stress "the heart of mathematics - its structure and meaning" (Kramer, p. 5) - which has been neglected by both an approach based on repetition and one based on incidental learning.

For the past twenty years, much has been done to emphasize meaningful teaching with a greater emphasis on the "whys" of the processes as related to computational procedures. During this period, elementary school mathematics textbooks have reflected this trend. "As one examines current mathematics textbooks, teacher's manuals, and materials from some of the new curricula, one word clearly stands out - 'discovery'" (Kramer, p.81). The word itself, however, has different connotations in different programs, depending on the educational theory of which it is illustrative. A controversy seems to arize with the question of how much and what kind of guidance ought to be provided to the pupils in a learning situation. Followers of Jerome Bruner, for example, will advocate minimal teacher guidance and maximal student opportunity for trial and error, whereas, followers of Robert Gagnė support a more guided learning position which emphasizes the importance of careful sequencing of instructional experiences (Kramer, p.81).

These new insights gained from humanitarian educators, psychologists, and mathematicians have suggested that educators reexamine their goals of mathematical instruction. With the awareness of the need for new trends in educational development, one can begin to comprehend the significance of new educational designs. If there were one thing that today's educators are in unanimous agreement on, it would be that education must provide for individual differences and that personal needs must be considered along with social ones. This fact suggests that organizational change is necessary. Akin to organizational change is curricular change, bringing about a diversification of educational patterns.

The teacher, as an educator, is confronted with the question of identifying her role in the work of initiating and developing such changes. The teacher, unlike other types of educators, realizes her unique position in having the opportunity of implementing, herself, any changes that she develops. Her approach to meeting the challenge of educational reform is characteristically more pragmatic in nature than those educators whose purpose is to develop educational theories for others to utilize. Although her training and experiences may be limited, the teacher recognizes that she is perhaps better equipped to take the "ideal" program and adjust it to the practical situation of the classroom. Of course any deviation from the "ideal" will fall short of the educator's expectations. Thus the continuum of educational reform is maintained. It is not the purpose of this paper to present an "ideal" arithmetic program. The purpose of this paper is to attempt to illustrate how one teacher can use a knowledge of the foundations of a subject (mathematics), insights from psychological and educational research, and her own educational philosophy to bring about a new workable pattern of instruction that can be used under prevailing methods of school organization, and with instructional materials that are readily available.

CONSIDERATIONS IN PLANNING A MATHEMATICS CURRICULUM

In constructing a pattern of mathematics instruction, the teacher's first major objective is to clearly identify some major aims and purposes for development. This identification is usually aided by curriculum materials and specified school curriculum guides utilized by the particular school district. However, the goals defined in these sources are characteristically broad in scope. Although this factor conveniently provides for flexibility, it also necessitates the construction of goals that are more well defined so as to be of use in any operational sense.

In defining the major goals of a program, the teacher is confronted with the fundamental question of "Why teach mathematics?" Educators such as Guy Wilson and Leo Breuckner have answered this question by stressing the need for social usefulness. Wilson, in fact, sees the need for teaching mathematics as identified with "its use as a simple tool in business" (DeVault, p.7). Other educators, however, are not in complete agreement. Edward G. Begle, director of the School Mathematics Study Group (S.M.S.G.), does not see the purpose of mathematics instruction as an effort to build up mathematics just for its applications. He views the use of mathematics in our society as a growing endeavor, with yet unknown possibilities. In developing the S.M.S.G. program, Begle and his associates made careful consideration of this fact in setting down their goals.

> We felt it likely that a considerable number of our students would have to learn within their lifetimes some new mathematics not yet in existence

> when they were in school, and we felt that having a good understanding of mathematics would make the learning easier than would the mere possession of technical skills (Eisner, p.71).

Most educators then, are in agreement that mathematics instruction is essential for meeting future needs; they differ, however, in their evaluation of what these needs are. The teacher will be able to define the major goals of instruction only after these needs are determined.

The degree to which one views the need for mathematics instruction in light of social usefulness, and/or as a tool for developing educated individuals who possess the capabilities of extending existing knowledge to new areas of growth, will in turn affect the selection of content material. Some programs (Minnesota School Mathematics and Science Project (Minnemast) in particular) have correlated the study of mathematics with science in an attempt to capitalize on the "practical aspects of math" (Suydam & Riedesel, p.146). Others (Madison Project in particular) have made use of a math lab approach which seeks an effective way of presenting mathematics as a matter of discovery. But regardless of what basic approach is utilized, most innovators of current curricula reform are consistent in the desire to expand and modify the scope and sequence of traditional programs. Robert Davis, director of the Madison Project, believes that "young children can learn far more than the 'traditional' school program attempts to teach them" (Kramer, p.225).

Yet Piaget has suggested that a child's learning of mathematics occurs at several levels which are clearly defined. The question of what content is appropriate to effective learning is clearly implied in his research findings. With the acceleration of the introduction of certain topics into the curricula, many of the new mathematics programs have been the target of severe criticism. David Rappaport is among those who are criticizing the schools for trying to present "too much too soon". He writes,

> It is my contention that this new emphasis has introduced concepts and practices in violation of sound principles of learning theory. ••• I believe that the introduction of highly sophisticated mathematical concepts in the early elementary grades will take the child away from the meaningful understanding of basic arithmetic concepts needed in his everyday, practical experiences and also in his preparation for further learning in other areas (Vigilante, pp.417, 420).

Although the implications of Piaget's theories for mathematics education are far from being realized, a fundamental question that arizes is whether or not it is possible to accelerate the course of development of the child through the various stages. It has been a common viewpoint among many of the new mathematics programs that this acceleration can be attained.

> Although children think and reason in different ways, they all pass through certain stages depending on their chronological and mental ages and their experiences. We can accelerate their learning by providing suitable experiences, particularly if we introduce the appropriate language simultaneously (Schools Council for the Curriculum and Examinations, p.9).

In 1966, Muller-Willis investigated the possibility of accelerating children's development while using some of Piaget's original experiments. His findings showed that "children who had studied the Minnemast materials were significantly better than the control group of kindergarteners who had studied in conventional programs" (Kramer, p.79). Another study by Nathan Gottfried in the following year, showed that "over twice as many children in Minnemast, as compared with conventional programs had attained these various conservation concepts [as defined in Piaget's stages of intellectual development] by the end of kindergarten" (Kramer, p.79).

Other research reports, however, will contradict these findings and support the theory that acceleration cannot be attained through instruction. Perhaps such factors as the amount of time spent on instruction, the social interaction with other children (absent from Piaget's research where children were questioned individually in a laboratory setting), and cultural milieu may be significant factors causing opposite research results (Kramer, pp.79-80).

Regardless of what viewpoint is taken, however, researchers have convinced educators that knowledge concerning the psychology of learning is an important aspect to consider in planning a curriculum. But even if success is attained in presenting new material at an earlier level, the fundamental question must still be raised, "Does the fact that content <u>can</u> be taught solve the problem as to whether it <u>should</u> be taught?" (Deans, p.98).

The teacher in selecting the math content for the curriculum must consider what can be taught at that level, and then narrow the selection by judging what should be taught. Several factors should be considered:

1) Present and future mathematical needs of children.
2) The value of the content from the standpoint

of interest and motivation.

3) The extent to which it improves the acquisition of knowledge and skills which are deemed essential in the regular program.

4) Whether it offers methods and content of promise in promoting reasoning, thinking, and concept development (Deans, p.98).

Examples of new content, and the expansion of several topics already recognized as essential parts of a traditional program are

> prime numbers, composite numbers, factors, exponents, different numeration systems, properties of numbers, additional terms and symbols, algebraic equations, inequalities, mathematical sentences and frames, number patterns, the number line, logic, simple probability, sets, graphing, geometry, and modular arithmetic (Kramer, p.169).

This wide selection of new topics enables the teacher to provide an enriching and diversified program, one which provides "children of different abilities with content appropriate to their intellectual level" (Deans, p.99).

Once the scope of the program is realized, the teacher is faced with determining the sequence that the instruction will follow. In order to do this effectively, some consideration must be made to the relationship that exists between the psychology of learning and methods for instruction.

The learning by discovery approach advocated by Jerome Bruner suggests a type of instructional technique which allows for much pupil-oriented activities. Bruner describes the child as moving through three levels of understanding during this process of discovery. The first level is identified with the child's manipulation of materials. The second level involves bringing the child to the level of understanding where he is thinking of objects but not manipulating them directly, and finally the child moves to the symbolic level where he is manipulating symbols and not mental images of objects. But discovery is not acquiring entirely new information, because "discovery, for Bruner, is a reorganization of something the discoverer already had" (Kramer, p.84).

Whereas Bruner describes learning as a somewhat intuitive process, Robert Gagne views learning as a transfer of training. For Gagne, in an instructional program, the child is carefully guided. Gagne follows a task analysis approach. His model of learning involves a "complex pyramid of prerequisites to prerequisites to prerequisites to the objective which is the desired capability" (Kramer, p.85).

These two contrasting theories have significant implications as to how a program of instruction is sequenced. The sequences of the curriculum growing from these two positions would be quite different. A curriculum based on Bruner's work would, for example, begin with problem-solving and work its way down to the lower levels which involve such things as principles, concepts, and the facts. A curriculum based on Gagne's ideas, would work just the opposite. The sequence would begin with the lower levels, with instructi n centering on the "prerequisites" and gradually move upward to the more complex tasks. "For Gagne, the sequence is from the simple to the complex; for Bruner, one starts with the complex and hopes to learn the simple components in the context of working with the complex" (Kramer, p.91).

Many new programs are experimenting with instructional designs which favor one of these two theories. Programs such as the Mathematics Project of Sherbrooke, the Madison Project, and Minnemast, which stress discovery learning approaches and use math lab materials or other manipulative devices can be associated with Bruner's viewpoints. Programs such as the Cambridge Conference on School Mathematics, the Stanford Computer Assisted Instruction Project, and S.M.S.G., or projects which make use of programmed materials or place a significant stress on behavioral objectives, could be classified with Gagne's thought.

The teacher, therefore, must first identify her learning theories before she can adopt an instructional style. She is then free to sequence her method of instruction within the structure of the system of mathematics itself.

> Most certainly, mathematical structure must play a central role in curriculum reform. Schools cannot teach contemporary mathematics unless the curriculum reflects the things that contemporary mathematicians are interested in. However, once having decided upon the mathematical structure that seems the most appropriate for the schools, there remains the very difficult problem of structuring the sequence of ideas from grades K through 12 so as to make the mathematical structure easily understood (Vigilante, p.78).

Mathematics is a highly structured discipline following a highly logical order. "The conceptual structure of a body of knowledge is nowhere better defined than in mathematics" (Frost & Rowland, p.252). The correct sequencing of skills is therefore highly dependent on the teacher's knowledge of the structure of mathematics. However, this logical sequencing of skills, defined by the structure of the discipline, does not always follow the most sound psychological principles governing learning. For example, using the structure of the mathematical system, a mathematics curriculum could be sequenced so that the learning of the multiplication tables would begin with 1 x 0 and progress to 12 x 12. However, taking into account psychological research on memory and learning, educators have suggested the sequencing of learning the basic multiplication facts in families as this provides for easier memorizing and greater understanding.

The sequencing of skills, therefore, is far from a simple task. The teacher, in preparing a mathematics curriculum, must consider the logical structure of mathematics and combine this knowledge with what is known through research in educational psycholcgy.

Once the scope and sequence are determined, an organizational plan for instruction must be adopted. Emphasis has already been given to the importance of exploring mathematical content in light of its appropriateness for different levels of ability. The need for providing for individual differences should also be a primary consideration in selecting a pattern of organization. Usually classroom organization is governed by school organization. Nongraded programs with team-teaching situations provide different work structures, for example, than do departmentalized graded programs. Yet, the fundamental problem of how to further subgroup pupils in the classroom exists under any program that falls short of complete individualization. "Even when the organizational pattern of the school facilitates the teacher's task, teachers still find that some grouping within the class is necessary" (Deans, p.14).

Differentiating the mathematics program to operate under prevailing methods of school organization, and with instructional materials that are readily available usually follows one of two types: a program of varied pace, or a program of varied difficulty (Vigilante, p.88). In a program of varied pace, the course work is sequential and aproximately the same content material is covered by each pupil. The rate of learning, however, is different for each child. Bright pupils advance through the program more quickly than do pupils of lesser ability. Homogeneous forms of grouping are often initiated for this type of program, although heterogeneous classes often use a subgrouping that involves an accelerated groups, a conventionally paced group, and a slow moving group. Individualized Programmed Instruction and forms of "contract learning" are also used to provide for individual differences by varying the pace of instruction.

The School Mathematics Study Group is one of the more widely known programs that foster such a system of organization. E. G. Begle, director of the program, believes that "adjusting the speed to the students in a capacity" (Suydam & Riedesel, p.76) is a good way of handling individual differences. Although it is stressed that S.M.S.G. does not advocate any one instructional approach, the use of their programmed materials would probably work best in a situation where pupils were permitted to work at their own rate.

In a program of varied difficulty or depth, a variety of approaches to the learning of a topic is presented. Rolland Smith describes this concept as "levels of learning" which "recognizes the fact that children of various degrees of ability or maturity can learn to do the same thing on several different levels from the simple concrete to the more complex abstract" (Vigilante, p.90). Under this form of organization, the whole class moves from topic to topic together, with abler students advancing to extension or enrichment topics while less able students continue to use manipulative devices to grasp the understanding of the concept. Grouping under this organizational plan is more flexible with groupings changing from topic to topic. "The sequence of topics must be carefully planned and adequate practice must be provided. Of greatest importance, the materials used should offer a variety of challenges" (Vigilante, p.90).

One such program that utilizes an organizational technique which varies the difficulty of instruction is A Systems Approach to Improving Mathematics Instruction (SAM), an experimental program for fourth grade pupils, originating in the Pittsburgh area. "Through a systematic yet flexible procedure, the teacher proceeds from performance level to performance level with the entire class at a pace he believes in keeping with the ability, interest, and achievement of his pupils" (SAM Manual, p.3).

The teacher in selecting a pattern of organization, must consider the over-all organizational plan of the school. Considering the time and materials available to her, she can then choose a plan of operation that will best facilitate the grouping of students to meet individual differences.

In setting up a mathematics curriculum, then, the teacher must be aware of many considerations. She must set down long-range objectives based on the educational philosophy of the school district, which is hopefully her own. She must have the necessary discretion to determine what topics can and should be taught. She must then use what is known about the structure of the mathematical system, as well as educational psychology to sequence these topics for instruction. The teacher can then plan a practical organizational pattern that will operate well under prevailing school organizational plans and with the materials that are readily available.

AN INDIVIDUAL APPROACH TO PLANNING A MATHEMATICS CURRICULUM

Major Objectives

In selecting the primary objectives for a fifth grade mathematics curriculum, I carefully considered the purposes for teaching mathematics in light of identifying the needs of the students. Although I recognize the significance of Wilson's claim that the arithmetic program should provide purposful instruction, useful in daily life, I question his interpretation of "useful" and doubt the value of attempting to limit the aims and purposes for teaching arithmetic to one major goal. I agree with Begle's foresight concerning the future use of mathematics and strongly recognize the need to provide a program which will attempt to equip the student with the capability of extending his knowledge to new areas of growth. In attempting to speculate on the present and future needs of students, however, it is essential to realize that these needs will vary according to the individuality of each child. Educators can very easily limit a child's mathematical potential by choosing one set of goals aimed for the "average" student. Determining the mathematical needs of most students can be justifiable for the majority, but the question of how well the program will meet the needs of the exceptional student must also be raised. This is not to say, however, that the entire mathematics program should be geared for the future scientists and pure mathematicians. Selecting goals founded on the needs of a minority is perhaps less desirable. The elementary school arithmetic program must include learning activities involving the four basic processes and also provide other interesting phases of mathematics that will help prepare those students who are interested and able, to continue in new mathematical emphases at later educational levels (Frost & Rowland, p.337). One major criterion I adopted, therefore, was that the set of major goals used in developing a mathematics program should include provision for meeting individual needs and capabilities.

Edwina Deans seems to meet this criterion as well as maintain the desire for a well balanced approach to the inclusion of socially useful arithmetic and pure mathematics. She has listed the following goals as characteristic of what is being reflected in current mathematics curricula.

1. To develop concepts of quantity and of quantitative relationships; to develop the child's ability to think in quantitative situations.
2. To develop as high a level of skill in computation as is realistic in consideration of each child's potential.
3. To recognize those situations in daily living requiring mathematical solutions and the appropriate techniques for solving them.
4. To develop an understanding of our numeration system and to recognize the value of base 10 [ten] in concept development as children work with processes.
5. To help each child understand the structure of mathematics, its laws and principles, its sequence and order, and the way in which mathematics as a system expands to meet new needs.
6. To help each child prepare for the next steps in mathematical learning which are appropriate for him in terms of his potential and his future educational requirements (Deans, p.4).

These goals do not forsake the former objectives of traditional programs, but broaden them to include new perspectives. I have selected this set of major objectives as my major goals in writing a fifth grade curriculum.

Selecting Content

In selecting the mathematical content for a fifth grade class, I carefully surveyed various textbooks and published materials from recent mathematics projects to determine the scope of work usually presented during the fifth year of instruction. For the most part, inclusion of the content in these sources has been justified through success in experimental situations. In selecting appropriate content, however, it is important for the teacher to be aware of the past instruction of her class and to be somewhat cognizant of the range of ability levels of her students. For this reason, I paid considerable attention to the presentation of material from the textbook series in use at my school. On the basis of past experience, I carefully included topics usually found difficult by the majority of fourth graders and avoided topics usually presented in the sixth grade.

If individual needs and abilities are to be considered, some differentiation of subject matter must be made. Surveying the topics I felt could and should be presented (based on the factors for consideration listed by Deans), I selected those topics which I felt were important for the majority of students to acquire, and listed them under "Basic Levels". The majority of these topics concerned the use of the four basic arithmetic operations with whole numbers and fractions. The remaining topics were considered for enrichment work, presented to those students who would acquire the knowledge of the material covered in the basic levels at a faster pace than those students who did not. Therefore, the extent of content presented to each child would be individually determined by the child's performance. The majority of these topics included areas of mathematics other than arithmetic, which might foster reasoning and thinking skills, and an interest in the future study of mathematics as a discipline.

Sequencing Content

In determining a sequence for instruction, I carefully considered a possible logical order of skills which would adhere to the structure of mathematics and foster psychological principles of learning. In presenting mathematics as a structured system, I planned for most topics to be developed and pursued until a certain level of understanding is achieved. Every possible effort, however, was made to provide activities that would relieve the intensity of certain long sections. For example, units of geometry and measurement are interjected between levels involving arithmetic operations. Although the organizational pattern of my curriculum, which is explained in a later section, necessitated a non-sequential order of enrichment topics, topics for enrichment were selected to help enhance a variety of learning situations and create interest in new areas of mathematical reasoning.

The instructional approach I adopted adheres more to Gagne's model of learning than to that of Bruner's. It has been pointed out that "teachers who have tried out the new materials are finding that developing concepts and helping children build a mathematics structure require continuous guidance" (Deans, p.12). Based on this viewpoint that formal guidance is necessary, I selected a sequence that advocates an instructional program of guided learning, rather than one of intutitive discovery. Instruction was planned to begin with the simpler ideas and advance to the more abstract concepts. I therefore made consideration of prerequisite skills needed for the development of topics. The use of behavioral objectives facilitated the organization of the subject matter in a hierarchical form.

Organizing Instruction

In selecting the major objectives for writing the curriculum, I adopted an educational philosophy that placed high priority on providing for individual differences. The need for a program to provide some social interaction among children, however, should also be realized. Therefore, in planning a pattern of organization, I sought a way of grouping students that would develop individual capabilities and yet provide an opportunity for pupils to work together.

In order to insure a flexible method of grouping, I avoided an organizational plan which grouped children heterogeneously on the basis of ability. A program which varied the amount of content and degree of difficulty by providing a variety of approaches to the learning of a topic was preferred. The systems approach to mathematics instruction developed by the SAM experimental project offered a pattern for instruction that could be adapted to normal classroom use, provided the teacher is willing to try her hand at writing some self-instructional materials each week for a group of her students. Instruction in the SAM program follows a guided learning approach, using a method of grouping based on the individual's mastery of the objectives for each basic level of instruction.

Although instruction of a level may require anywhere from two to seven days, depending on the class's needs, most levels require five periods of instruction. The five period cycle follows this general procedure. On the first day, the whole class is presented with the new concepts (usually two behavioral objectives) to be achieved by all or most students. A short film presentation is used for motivational purposes followed by the teacher's presentation. During the second period, a performance sheet, to be used for grouping procedures, is administered to all pupils. The performance sheet breaks the learning tasks into the two behavioral objectives and makes possible the separation of pupils into groups according to their achievement per behavioral objective. The remaining time of the second period is used to review the work used for "maintenance" activities for the week. The use of maintenance activities is to insure competence in past levels and provide opportunities for reteaching of past concepts when necessary.

Prior to the third period, the teacher uses the results from the performance sheets to separate into groups those children who did not meet the goals for Objective #1 and Objective #2 (Group 1); those pupils who accomplished the goal for Objective #1 but did not meet the goal for Objective #2 (Group 2); and those pupils who mastered both objectives (Group 3). During the third period, the pupils are given differentiated instruction according to their needs as exhibited on their performance sheet. While Group 3 works independently on maintenance and reinforcement work, the teacher is free to reteach Groups 1 and 2. While Group 2 works on maintenance activities, the teacher re-instructs Group 1. This instruction is followed by practice work on that specific objective (1) which enables the teacher to use the remaining time of period 3 in reteaching the concept presented in Objective #2 to Group 2. During period 4, Group 3 proceeds on their own, using self-instructional enrichment/extension work. Group 2 begins work on basic practice worksheets for Objective #2 while the teacher spot-checks Group 1's work and proceeds to reteach the concept of Objective #2 to Group 1. Group 1 uses the rest of the period to work on the basic worksheets of Objective #2 while the teacher is free to spot-check the work being done by Groups 2 and 3 and provide individual help where needed. During the fifth period, each group should be completing its appropriate work and time is given for the administration of a second performance sheet to Groups 1 and 2 to determine if mastery of the objectives was accomplished. The self-instructional materials also provide an evaluation sheet for Group 3. The scores from the second performance sheet will indicate whether it is necessary to spend an additional day or two on the level and also indicate which pupils will need review and reteaching which can be accomplished at later levels during the times the teacher is free to help individuals.

Variety is added to the system's instructional procedures when at certain levels, the teacher instructs Group 3 students on the third through fifth day and self-instructional work is given to those students who have not met the behavioral objectives. This procedure occurs with basic levels which fall outside the realm of the basic arithmetic operations. These levels often constitute a repeat of a previous enrichment/extension topic, and the self-instructional work that was used for the enrichment group (Group 3) at the former level, is now given to those students who failed to meet the objectives.

Although the operation of this systems approach to instruction is dependent on the use of the SAM published materials, a teacher can utilize this organizational structure by adapting what materials are readily available in the classroom and supplementing these with teacher-made activities. Appropriate textbook pages can be assigned for maintenance and reinforcement work. When students record their work in a notebook, checking off the assignments as they are completed will enable the students to work at their own pace. Several copies of teachers' editions should be available for pupil-checking, since the completion of these activities will be accomplished at an individual rate. Performance sheets are easily written since they are directly taken from the behavioral objectives. Their use as a device for initial and terminal evaluation is also readily recognized. Their construction, therefore, is a worth-while endeavor for evaluation as well as grouping procedures. Objective #1 and #2 worksheets are simple practice pages which provide similar problems to those found on the performance sheets and are usually also easily written.

The construction of self-instructional materials is definitely a challenge to a teacher. However, using the various resources, such as other math texts and professional teacher magazines, which are available in most schools or professional libraries, facilitates this task. Writing self-instructional materials not only provides enriching learning experience for the pupils, but also provides the teacher with the opportunity to enhance her knowledge of other areas of mathematics. Sequencing the content for a self-instructional mode of learning necessitates the teacher's thorough knowledge of the topic. In this sense, self-improvement in the learning of mathematics can be accomplished by both pupils and teacher. Since instruction of each level usually takes one week, the teacher can initially accomplish the task of providing self-instructional materials for each level by writing a set of materials each week. If this pace is maintained, the task becomes realistic.

Although a certain organizational pattern is adopted, it should be flexible to change. The fact that the curriculum is teacher-constructed should enable it to be easily modified to suit the individual class's needs when necessary.

ADDITION OF FRACTIONS II

A. <u>Behavioral Objectives</u>

1. Given an addition example involving two proper, unlike fractions, the pupil finds the least common denominator, converts the fractions to equivalent forms, and computes the sum giving the answer in simplest terms.

2. Given an addition example involving three proper, unlike fractions, the pupil finds the least common denominator, converts the fraction to equivalent forms and computes the sum giving the answer in simplest terms.

B. <u>Procedures</u>

1. Period One (Whole Class)

a. Motivational Technique

Materials Needed: 3 large containers of blue, red, and yellow colored water; measuring cups; 3 empty containers.

Relate this tale: As I look around the class today, I notice an array of many colors. But do you know at one time, in a far-away kingdom, the things that men made consisted of only three colors? Everything was either red, blue, or yellow. Now the king of this land was very unhappy. He dreamed of fancy robes the colors of the morning sun, the tall-blowing grass, and the majestic mountains. So he called his wisest advisors together and offered them a special reward if they could find a way to make his dream come true. After many nights of work in his laboratory, Merlin, the king's magician, discovered these secret formulas. I'll write them on the board. (Write: 1/4 cup yellow + 1/8 cup red = A
1/2 cup blue + 1/3 cup yellow = B
2/3 cup blue + 3/4 cup red = C)
We're going to do these experiments now to test out Merlin's magic. (Have different children come up and take turns measuring out the liquids and mixing them together.) Now the king was very happy, and he wanted his tailors to get to work immediately. But to make the special dyes for his silk robes, the tailors needed 1 cup of dye. After our lesson today, we should be able to see if these measurements will give us enough dye to make the robes for the king.

b. Draw a number line on the board, dividing the unit measurement into eighths. Ask for a volunteer to use the number line to show what 1/2 + 1/8 is equal

SAMPLE MATERIALS

In order to further demonstrate the content and organization of my curriculum, I have selected certain portions for exemplary inspection. I have included the "Curriculum Suggested Pace" for my fifth grade curriculum which lists the basic levels of instruction as well as the suggested enrichment topics for each level. The behavioral objectives for each basic level are also included. Teacher-constructed materials for two levels, one level for which the teacher instructs the students who need additional time for mastery of the objectives and one level in which the teacher instructs the enrichment group, are provided for exemplification. In order to facilitate an understanding of the operational structure of the curriculum, I have also included the complete lesson plan for these two levels.

to. Continue with other examples, such as 1/4 + 3/8 etc., until the majority of students are able to deduce that like fractions with common denominators are necessary to solving the sum.

c. Review methods for finding a common denominator, ways to write equivalent fractions, procedures for reducing fractions to lowest terms, and methods of simplifying improper fractions.

d. Combine these skills in solving addition problems involving two or three proper, unlike fractions. Wrap up the lesson by having pupils use the values in "Merlin's formulas" to determine if the sums will exceed 1 cup. It might be interesting to ask the students what the tailors might do with the two formulas which have sums that do not exceed 1 cup.

2. Period Two (Whole Class)

a. Administer performance sheet.

b. Pupil scoring (optional)

c. Teach review work to be used in maintenance activities. (Suggested topics: reducing fractions to lowest terms, and writing equivalent fractions).

d. Have pupils survey selected maintenance activities from textbooks. (Page 110 in Ginn Series and pages 190-191 in Addison Wesley may be used for this week's maintenance work).

Planning for Differentiated Instruction

Prior to period three, separate pupils into groups according to their achievement per behavioral objective. All pupils missing Objective #1 will constitute Group 1. Group 2 will consist of all pupils who have missed the second objective, but not the first. Group 3 will consist of all pupils meeting both objectives.

3. Period Three

a. Assign pupils to appropriate groups.

b. Groups 2 and 3 begin maintenance activities.

CURRICULUM SUGGESTED PACE
AND
BEHAVIORAL OBJECTIVES

c. Teach Objective #1 to Group 1. It is suggested that time be given to reviewing and reteaching methods for finding a least common denominator and ways of writing a fraction in equivalent forms. Use number lines and pictures initially to insure understanding. After aproximately 15 minutes of instruction, have pupils begin work on Objective #1 worksheets.

d. Pupils in Group 3 work on reinforcement exercises. (Pages 188-189 in Ginn may be used).

e. Teach Objective #2 to Group 2. It is suggested that the least common multiple method of finding the least common denominator is used. Provide students with individual help at the board for finding the sums of 3 proper, unlike fractions.

4. Period Four

* Behavioral Objectives for Enrichment Work

1. Given a practical situation involving tossing a coin or rolling dice, the child can list all the possible combinations of a given outcome.

2. Given a practical situation involving tossing a coin or rolling dice, the child can determine the probability of an event occurring and write it as a fractional numeral.

a. Group 1 continues Objective #1 worksheets or maintenance work.
Group 2 begins work on Objective #2 worksheets.
Group 3 - During the first five minutes, spot-check reinforcement work. Distribute self-instructional worksheets and provide brief introduction to this week's work on probability (May, pp.62-63). Have pupils begin work.

b. Spot-check Objective #1 worksheets with Group 1. Teach Objective #2 to Group 1. For teaching suggestions, see Period Three, e. Distribute Objective #2 worksheets to Group 1 and have pupils begin work.

c. Spot-check Objective #2 worksheets for Group 2. Explain reinforcement work (Ginn, pp.188-189). Have pupils begin work on these problems.

CURRICULUM SUGGESTED PACE

BASIC LEVELS	ENRICHMENT TOPICS
September	
* 1. Geometry I	Time Zones
* 2. Basic Facts (+ and -)	Magic Squares
* 3. Place Value I	Other Number Systems - Roman, Greek
* 4. Place Value II	Work in base IV
* 5. Addition	Functions
October	
6. Subtraction I	Sets - $\cup$ and $\cap$ with Venn Diagrams
7. Subtraction II	Negative Numbers
8. Problem Solving I	Multiple-step Problem Solving (+ and -)
+ 9. Measurement I	Addition and Subtraction of Inches & Feet
10. Basic Facts (x and ÷)	Perimeters
November	
11. Distributive Principle	Arithmetic with Frames
*12. Multiplication I	Problems in Fahrenheit Temperature
*13. Zero Pattern	Number Line Game (Add./Wes. pp.34-37)
*14. Multiplication II	Exponential Notation
15. Estimation	Estimating Square Roots
16. Multiplication III	Checking Multiplication by Casting Out 9's
December	
+17. Geometry/Measurement II	Constructions of Angles with Compasses
*18. Division I	Slide Rule
19. Division II	Discovering Number Patterns (factors of 3,5)
*20. Introduction to Fractions	Constructing Fraction Games
January	
21. Division III	Geometry-"How Can I?" (Arith. T. Nov. '70)
22. Division IV	Finding Surface Areas of Rectangular Prisms
23. Problem Solving II	Problems with Time, Rate, Distance
*24. Factors & Multiples I	Number Combination Game (Arith. T. Dec. '70)
*25. Factors & Multiples II	Fraction Bingo (Arith. T. Mr. '70)-Discovery
26. Equivalent Fractions	Euclidean Algorithm/Simplifying Fractions (Arith. T. Dec. '70)
February	
*27. Improper Fractions	Finding Averages
28. Inequalities	Cartesian Plane
+29. Geometry/Measurement III	Informal Geom. with milk cartons (Arith. T. Oct. '66)
*30. Addition of Fractions I	Clock Arithmetic
31. Addition of Fractions II	Probability
March	
32. Subtraction of Fractions I	Temperature/Changing from C. to F.
33. Sub. of Fractions II	Fraction Magic Squares
34. Problem Solving IV	Charts and Graphs
*35. Place Value III	Extending Decimals to Hundred Thousandths
*36. Multiplication of Frac. I	Mirror Lines - Symmetry
37. Multiplication of Frac. II	Using Fractions in Measurement (Add./Wes. pp. 220-21)

* indicates those levels which will probably require less than one week to complete.

\+ indicates those levels for which the teacher works with the enrichment group, rather than with those grouped for additional instruction.

d. Spend remaining time spot-checking all groups, giving individual help as needed.

5. Period Five

a. Group 1 continues with Objective #2 worksheets and maintenance work.
Group 2 continues with reinforcement and maintenance work.
Group 3 continues with self-instructional worksheets. Children in any of the groups who complete their work early are given the opportunity to play with one of the classroom math games.

b. Spot-check Objective #2 worksheets with Group 1. Help individuals who are still having trouble. Incomplete maintenance assignments from past levels can be done when enough practice has been given on the present level's work.

c. Provide individual help to pupils in Groups 2 and 3 as needed.

d. Fifteen minutes before the end of the period, administer Performance Sheet 2 to Groups 1 and 2. Collect papers when pupils are finished to use for evaluation purposes.

e. Collect Enrichment-Recall Sheet from Group 3 to be used for evaluation purposes.

CURRICULUM SUGGESTED PACE (Con.)

BASIC LEVELS		ENRICHMENT TOPICS
April		
*38.	Multipl. of Fract. III	Ratio
*39.	Division of Fract. I	Construction of Plane Figures with Compass
40.	Division of Fract. II	Pythagorian Theorem
41.	Problem Solving V	Complex Fractions
*42.	Problem Solving VI	Using Tables & Charts in Word Problems
May		
+43.	Geometry/Measurement IV	Topology
+44.	Cartesian Plane	Graphing Functions
+45.	Negative Numbers	Negative Numbers (+ and -)
+46.	Problem Solving VII	Problems in Simple Logic
47.	Weight Measurement	Multipl. & Division of oz., lb., and T.

June

Review and additional topics at the discretion of the teacher.

* indicates those levels which will probably require less than one week to complete.

\+ indicates those levels for which the teacher works with the enrichment group, rather than with those grouped for additional instruction.

PERFORMANCE SHEET 1

Addition of Fractions II

Name ______________________________

Date ______________

ADD.

1. $\frac{1}{2} + \frac{1}{5}$

2. $\frac{1}{4} + \frac{3}{8}$

3. $\frac{2}{9} + \frac{1}{4}$

4. $\frac{2}{3} + \frac{3}{4}$

5. $\frac{1}{8} + \frac{5}{12}$

6. $\frac{5}{6} + \frac{2}{3}$

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐

****

1. $\frac{1}{3} + \frac{1}{4} + \frac{1}{8}$

2. $\frac{1}{5} + \frac{2}{3} + \frac{1}{6}$

3. $\frac{2}{9} + \frac{1}{6} + \frac{2}{3}$

4. $\frac{3}{10} + \frac{2}{5} + \frac{1}{4}$

5. $\frac{1}{8} + \frac{1}{2} + \frac{3}{4}$

6. $\frac{1}{3} + \frac{1}{4} + \frac{5}{12}$

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐

TOTALS		
GOALS	5	5

BEHAVIORAL OBJECTIVES

Geometry I.
1) Given the representation of a point, line, line segment, ray, and plane, the pupil writes the appropriate geometric term.

2) Given a picture of a physical object, the pupil can identify a likeness between that and a point, line, line segment, ray, and plane.

Basic Facts (+ and -)
1) Given basic addition and subtraction examples which have sums or minuends of 9 or less, the pupil writes the sum or difference.

2) Given basic addition and subtraction examples which have sums or minuends of 10 through 18, the pupil writes the sums and differences.

Place Value I.
1) Given a numeral less than 1000, read and written in words, the pupil selects, from a series of four numerals, the appropriate numerical representation.

2) Given a numeral less than 1000, the pupil is able to supply the missing place value words when the numeral is expressed according to its place value components.

Place Value II.
1) Given a numeral less than 1,000,000, read and written in words, the pupil is able to select from a series of four numerals the appropriate numerical representation.

2) Given a numeral less than 1,000,000 the pupil is able to supply the missing place value words when the numeral is expressed according to its place value components.

Addition
1) Given 2 numerals less than 10,000, the pupil supplies the correct sum.

2) Given 3 to 5 numerals less than 10,000, the pupil supplies the correct sum.

Subtraction I.
1) Given a subtraction example with 3 or 4 digit numerals, where subtraction requires no regrouping, the pupil computes the difference.

2) Given a subtraction example with 3 or 4 digit numerals where subtraction requires regrouping from the ones' to the tens' place, the tens' to the hundreds', and/or the hundreds' to the thousands' place, the pupil computes the difference.

PERFORMANCE SHEET 2

Addition of Fractions II

Name ______________________________

Date ____________________

ADD.

1. $\frac{3}{4} + \frac{1}{8}$

2. $\frac{2}{3} + \frac{1}{6}$

3. $\frac{3}{4} + \frac{2}{9}$

4. $\frac{1}{3} + \frac{1}{4}$

5. $\frac{1}{6} + \frac{7}{8}$

6. $\frac{1}{5} + \frac{3}{10}$

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐

****

1. $\frac{2}{5}$, $\frac{1}{6}$, $+ \frac{1}{3}$

2. $\frac{1}{8}$, $\frac{1}{2}$, $+ \frac{1}{3}$

3. $\frac{1}{6}$, $\frac{3}{8}$, $+ \frac{1}{3}$

4. $\frac{1}{6}$, $\frac{3}{4}$, $+ \frac{1}{12}$

5. $\frac{2}{9}$, $\frac{1}{6}$, $+ \frac{1}{3}$

6. $\frac{3}{8}$, $\frac{1}{12}$, $+ \frac{3}{4}$

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐

TOTALS		
GOALS	5	5

Subtraction II.
1) Given a subtraction example with 3 or 4 digit numerals, having a 0 in the tens' place of the minuend, requiring regrouping from the ones' to the tens' place, the pupil computes the difference.

2) Given a subtraction example with a 4 digit numeral which is a multiple of 1000 for a minuend, and a 3 or 4 digit subtrahend, the pupil computes the difference.

Problem Solving I.
1) Given an addition or subtraction statement problem, the pupil writes the name of the appropriate operation necessary to solve the problem.

2) Given an addition or subtraction statement problem, the pupil uses the appropriate operation to solve the problem.

Measurement I.
1) Given two line segments, pupils are able to use such measurement tools as rulers to determine if they are congruent.

2) Given a line segment, pupils are able to determine its length to the nearest $\frac{1}{2}$ inch.

Basic Facts (x and ÷)
1) Given open basic fact multiplication and division sentences involving factors, divisors, and dividends of the 2, 3, 4, and 5 number families, the pupil writes the correct products or quotients.

2) Given open basic fact multiplication and division sentences involving factors, divisors, and dividends of the 6, 7, 8, and 9 number families, the pupil writes the correct products or quotients.

Distributive Principle
1) Given a 3 or 4 place numeral, the pupil expands it; and given an expanded 3 or 4 place numeral, the pupil contracts it into a single numeral.

2) Given a 2 or 3 place factor, and a single digit factor, from 2 through 5, where multiplication involves no regrouping, the pupil expands the larger factor, multiplies each part separately by the single factor, and then adds the partial products together to find the final product.

Multiplication I.
1) Given a multiplication example with a 2 digit factor and a 1 digit factor, the pupil computes the product.

2) Given a multiplication example with a 3 digit factor and a 1 digit factor, the pupil computes the product.

OBJECTIVE 1
WORKSHEET 1

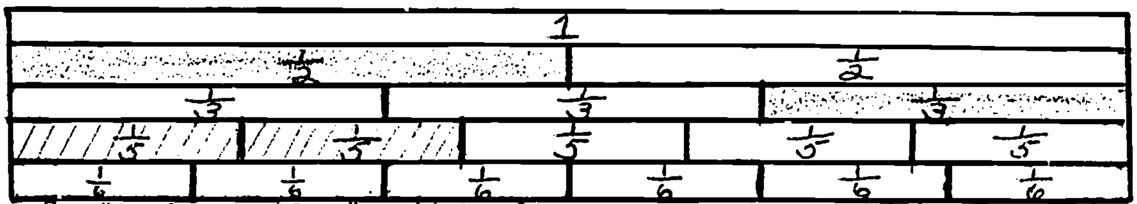
Addition of Fractions II

Name ______________________________

Date ______________________________

Susan wrote this problem on the board: $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$

Jane saw her work and said, "That doesn't make sense." She drew this picture.



"See, Sue", she said, " 2/5 isn't even as big as 1/2! It can't be what you have when you put 1/2 and 1/3 together."

Look at the picture; $\frac{1}{2}$ is the same as how many sixths? ________

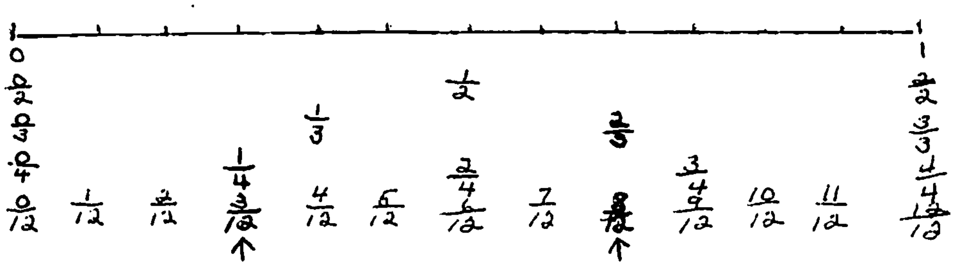
$\frac{1}{2} = \frac{\ }{6}$? $\frac{1}{3}$ is the same as how many sixths? ________

$\frac{1}{3} = \frac{\ }{6}$? $\frac{3}{6} + \frac{2}{6} = \frac{\ }{6}$? $\frac{1}{2} + \frac{1}{3} = \frac{\ }{6}$?

* Susan said, "To add fractions with unlike denominators, first find a common denominator."

Do you remember the ways to find a common denominator? Here they are!

1) Use a picture.



Now find a common denominator for $\frac{1}{4}$ and $\frac{2}{3}$. ________

$\frac{1}{4} = \frac{\ }{12}$? $\frac{2}{3} = \frac{\ }{12}$? $\frac{3}{12} + \frac{8}{12} = \frac{\ }{12}$?

2) Use sets of equivalent fractions. $\left(\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}\right)$
$\left(\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}\right)$

$$\begin{array}{rcl} \frac{1}{4} & = & \frac{3}{12} \\ + \frac{2}{3} & = & \frac{8}{12} \\ \hline & & \frac{\ }{12} \end{array}$$

Zero Pattern
1) Given two factors consisting of a single digit and/or multiples of tens or hundred, the pupil identifies the key fact (digits to be multiplied before affixing the zeros).

2) Given 2 factors consisting of a single digit and/or multiples of ten or hundred, the pupil writes the product.

Multiplication II.
1) Given a multiplication example with a two digit factor and a factor which is a multiple of ten, the pupil computes the product.

2) Given a multiplication example with a 3 digit factor and a 2 digit factor which is a multiple of ten, the pupil computes the product.

Estimation
1) Given a 2, 3, or 4 digit numeral, the pupil rounds the 2 digit numeral to the nearer ten, the 3 digit numeral to the nearer hundred, and the 4 digit numeral to the nearer thousand.

2) Given a statement problem involving addition, subtraction or multiplication, the pupil estimates the sum, difference, or product.

Multiplication III.
1) Given a multiplication example with a 2 or 3 place factor and another 2 place factor, the pupil computes the product.

2) Given a multiplication example with a 3 or 4 place factor and another 3 place factor, the pupil computes the product.

Measurement II.
1) Given 2 rays that form an angle, the pupil names the angle using letters or numbers and identifies it as right, acute, or obtuse.

2) Given two angles, the student identifies one angle as less than, greater than, or equal to the other in measurement, through observation and the use of protractors.

Division I.
1) Given an open basic division fact in which the quotient or one of the factors is missing, the pupil converts it to a known multiplication example and writes the missing numeral that will close the sentence.

2) Given a drawing of a set of objects which is divided into equal subsets, the pupil writes the division sentence that is appropriate for the set representation.

OBJECTIVE 1
WORKSHEET 2

Addition of Fractions II

Name ____

3) Find the least common multiple.

Multiples of 4 = (4,8, 12, 16, 20, ···)
Multiples of 3 = (3, 6, 9, 12, 15, ···)
L.C.M. = 12

$$\frac{1}{4} = \frac{1\times3}{4\times3} = \frac{3}{12}$$

What numeral would you replace the ? with?

$$\frac{2}{3} = \frac{2\times?}{3\times?} = \frac{N}{12}$$

______

Add these fractions. (Remember to find a common denominator and reduce your answers to simplest terms when possible.)

A. $\frac{1}{4} = \frac{1}{4}$

$+ \frac{1}{2} = \frac{\ }{4}$

$\frac{\ }{4}$

B. $\frac{1}{5} = \frac{3}{15}$

$+ \frac{1}{3} = \frac{\ }{15}$

$\frac{\ }{15}$

C. $\frac{1}{9} =$ ____

$+ \frac{2}{3} =$ ____

____

D. $\frac{1}{4} =$ ____

$+ \frac{1}{8} =$ ____

____

E. $\frac{1}{5} =$ ____

$+ \frac{3}{10} =$ ____

____ = ____

F. $\frac{1}{3} =$ ____

$+ \frac{1}{6} =$ ____

____ = ____

G. $\frac{2}{3} =$ ____

$+ \frac{4}{9} =$ ____

____ =

H. $\frac{11}{12} =$ ____

$+ \frac{1}{3} =$ ____

____ =

I. $\frac{5}{6} =$ ____

$+ \frac{2}{3} =$ ____

____ = ____ =

J. $\frac{5}{12} =$ ____

$+ \frac{3}{4} =$ ____

____ = ____ =

Division II.

1) Given a division example with a one-digit divisor and a 2 digit dividend, where the divison algorithm requires one multiplication step using the scaffold method, the student uses scaffolding to determine the quotient (with or without a remainder).

2) Given a division example with a one digit divisor and a 2 or 3 digit dividend where the division algorithm requires 2 multiplication steps using the scaffold method, the student uses scaffolding to determine the quotient (with or without a remainder).

Introduction to Fractions

1) Given an illustration showing a fractional part of a region, the pupil writes the appropriate fraction used to name the shaded region.

2) Given an illustration showing a fractioanl part of a set, the pupil writes the appropriate fraction used to name that part of the set.

Division III

1) Given a division example with an even tens' divisor where scaffolding requires one step, the pupil uses the scaffold method to determine the one-digit quotient (with or without a remainder).

2) Given a division example with an even tens' divisor where scaffolding requires two steps, the pupil uses the scaffold method to determine the 2 digit quotient.

Division IV.

1) Given a division example with a 2 place divisor, where scaffolding requires 1 or 2 steps, the pupil uses the scaffold method to determine the one or two digit quotient.

2) Given a division example with a 2 place divisor, where scaffolding requires 3 steps, the pupil uses the scaffold method to determine the 3 digit quotient (with or without a remainder).

Problem Solving II.

1) Given a multiplication or division statement problem, the pupil writes the name of the appropriate operation used to solve the problem.

2) Given a multiplication or division statement problem, the pupil uses the appropriate operation to solve the problem.

Problem Solving III.

1) Given an addition, subtraction, multiplication, or division statement problem, the pupil writes the name of the appropriate operation used to solve the problem.

Factors & Multiples I.

1) Given a composite number, the pupil writes all possible factor combinations.

2) Given any whole number, the pupil writes a specified number of multiples for that number.

OBJECTIVE 2
WORKSHEET 1

Addition of Fractions II

Name ______________________________

Date ______________________________

Use the least common multiple method for finding a least common denominator for these sets of fractions:

Example: $\frac{1}{4}, \frac{1}{6}, \frac{1}{3}$ Multiples of 4 = (4, 8, 12, 16, 20, 24, ...)

Multiples of 6 = (6, 12, 18, 24, 30, 36, ...)

Multiples of 3 = (3, 6, 9, 12, 15, 18, ...)

Least common multiple = 12 Least common denominator = 12

****

1) $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}$ Multiples of 2 = ( )

Multiples of 4 = ( )

Multiples of 8 = ( )

Least common multiple = ______ Least common denominator = ______

2) $\frac{2}{5}, \frac{1}{3}, \frac{1}{6}$ Multiples of 5 = ( )

Multiples of 3 = ( )

Multiples of 6 = ( )

Least common multiple = ______ Least common denominator = ______

3) $\frac{2}{9}, \frac{1}{6}, \frac{1}{4}$ Multiples of 9 = ( )

Multiples of 6 = ( )

Multiples of 4 = ( )

Least common multiple = ______ Least common denominator = ______

4) $\frac{1}{4}, \frac{7}{12}, \frac{1}{8}$ Multiples of 4 = ( )

Multiples of 12 = ( )

Multiples of 8 = ( )

Least common multiple = ______ Least common denominator = ______

5) $\frac{2}{3}, \frac{5}{8}, \frac{1}{6}$ Multiples of 3 = ( )

Multiples of 8 = ( )

Multiples of 6 = ( )

Least common multiple = ______ Least common denominator = ______

Factors & Multiples II.
1) Given any two numbers, the pupil determines the factors of each and then identifies the greatest common factor.

2) Given any two whole numbers, the pupil determines a sufficent number of multiples for each and then identifies the least common multiple.

Equivalent Fractions
1) Given a fraction in lowest terms, the pupil demonstrates his knowledge of equivalent fractions by supplying the missing numerator in an example where the denominator is given in an equivalent fraction of higher terms.

2) Given a fraction of higher terms, the pupil reduces it to lowest terms.

Improper Fractions
1) Given a mixed numeral, the pupil converts it to an improper fraction.

2) Given an improper fraction, the pupil converts it to a mixed numeral.

Inequalities
1) Given two proper fractions, the pupil is able to use the "cross-product demonstration of equivalent fractions" to replace the open frame with the correct sign, = or ≠.

2) Given two unequal proper fractions, the pupil uses a method of finding equivalent fractions to write the fraction with common denominators to compare them, and replace the open frame with the correct sign, < or > .

Geometry/Measurement III.
1) Given the dimensions of any of the following 4 sided figures: parallelogram, rectangle, square, or rhombus, the pupil finds the perimeter.

2) Given the dimensions of any of the following 4 sided figures; parallelogram, rectangle, square, or rhombus, the pupil computes the area.

Addition of Fractions I.
1) Given an addition example involving two proper-like fractions, the pupil computes the sum and reduces the answer to simplest terms.

2) Given an addition example involving mixed numerals, composed of like fractions, the pupil computes the sum and reduces the answer to simplest terms.

OBJECTIVE 2
WORKSHEET 2

Addition of Fractions II

Name ____

Directions: Find a common denominator, rename the fractions, add, then write your answer in simplest terms. Match your answers at the bottom of the page.

1) $\frac{1}{4}$ = ____

$\frac{1}{6}$ = ____

+ $\frac{1}{3}$ = ____

2) $\frac{1}{2}$ = ____

$\frac{1}{4}$ = ____

+ $\frac{3}{8}$ = ____

3) $\frac{2}{5}$ = ____

$\frac{1}{3}$ = ____

+ $\frac{1}{6}$ = ____

4) $\frac{2}{9}$ = ____

$\frac{1}{6}$ = ____

+ $\frac{1}{4}$ = ____

5) $\frac{1}{4}$ = ____

$\frac{7}{12}$ = ____

+ $\frac{1}{8}$ = ____

6) $\frac{2}{3}$ = ____

$\frac{5}{8}$ = ____

+ $\frac{1}{6}$ = ____

7) $\frac{2}{3}$ = ____

$\frac{1}{2}$ = ____

+ $\frac{1}{4}$ = ____

8) $\frac{1}{6}$ = ____

$\frac{2}{9}$ = ____

+ $\frac{2}{3}$ = ____

9) $\frac{3}{10}$ = ____

$\frac{2}{5}$ = ____

+ $\frac{1}{4}$ = ____

****

A.	$\frac{2}{3}$ ____	D.	$1\frac{1}{8}$ ____	G.	$1\frac{1}{24}$ ____
B.	$\frac{23}{24}$ ____	E.	$\frac{3}{4}$ ____	H.	$\frac{23}{36}$ ____
C.	$1\frac{1}{18}$ ____	F.	$\frac{19}{20}$ ____	I.	$1\frac{5}{12}$ ____

Addition of Fractions II

1) Given an addition example involving two proper, unlike fractions, the pupil finds the least common denominator, converts the fractions to equivalent forms, and computes the sum giving the answer in simplest terms.

2) Given an addition example involving three unlike fractions, the pupil finds the least common denominator, converts the fractions to equivalent forms, and computes the sum giving the answer in simplest terms.

Subtraction of Fractions I.

1) Given a subtraction example with two proper, like fractions, involving no regrouping, the pupil computes the difference and writes the answer in simplest terms.

2) Given a subtraction example with two like fractions, involving regrouping of a mixed numeral, the pupil computes the difference and writes the answer in simplest terms.

Subtraction of Fractions II.

1) Given a subtraction example with two proper, unlike fractions, involving no regrouping, the pupil finds the least common denominator, converts the fractions to equivalent forms, and computes the difference, giving the answer in simplest terms.

2) Given a subtraction example with two unlike fractions, involving regrouping of a mixed numeral, the pupil converts the fractions to equivalent forms with common denominators, regroups, and computes the difference, writing the answer in simplest terms.

Problem Solving IV.

1) Given an addition or subtraction problem involving fractions, the pupil writes the name of the appropriate operation used to solve the problem.

2) Given an addition or subtraction statement problem, involving fractions, the pupil uses the appropriate operation to solve the problem.

Place Value III.

1) Given a 1 or 2 place decimal numeral, read and written in words, the pupil selects from a series of four numerals, the appropriate numerical representation.

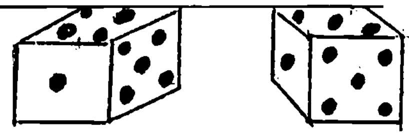
2) Given a 1 or 2 place decimal numeral, the pupil supplies the missing place value word when the numeral is expressed according to its place value components.

ENRICHMENT WORKSHEET

Probability

Name ______________________________

Date ______________________________



FOLD BACK HERE

"Take a chance." Have you ever tried your luck at the many dice and card games at a carnival? Were there some games that are easier to win at than others? How can you tell which games will give you a good chance of winning and which are the ones you will probably seldom win at? There is a type of mathematics that can help us with the answers to these questions. It is called PROBABILITY. Probability helps us to see how many times a certain thing will probably happen.

We have seen how fractions are used to tell about dividing a whole into equal parts. The mathematics of probability also uses fractions, but they are written to mean something different. See if you can do these exercises which will help you to figure out what are the odds of somethings happening.

---

1) Maria has a quarter. She and her friends like to try and guess whether a head or tail will turn up on a coin toss. Maria's friend, Ann, thought up another game. Ann said it would be harder to guess what would turn up on two coin tosses, rather than one. Four children played the game. Maria guessed that 2 heads would appear. Ann guessed that first a head and then a tail would be tossed. Joan guessed that first a tail and then a head would be thrown. Regina was the last to guess. What guess would she make to make sure that at least one of the children would guess correctly?

____

UNFOLD & CHECK

that 2 tails would be tossed

FOLD BACK

2) Joan thought of a way to write down the guesses so the children would not forget their guess. She used this short-cut: (H,H) means a guess of two heads.

What would a guess of two tails be written as? ________

(T,T)

To show Ann's guess Maria wrote (H,T) to mean a head would appear first, and then a tail. The girls made this chart. Can you finish the chart by writing the symbols they would use for Joan's and Regina's guesses?

Maria - (H,H) Ann- (H,T) Joan - ______ Regina - ______

UNFOLD & CHECK

Joan-(T,H)
Regina-(T,T)

Multiplication of Fractions I
1) Given a pictorial representation, the pupil can write the appropriate open multiplication sentence to represent the pictured concept.

Multiplication of Fractions II.
1) Given a multiplication example with two proper fractions, the pupil computes the product and writes the answer in simplest terms.

2) Given a multiplication example with at least one improper fraction as a factor, the pupil computes the product and writes the answer in simplest terms.

Multiplication of Fractions III.
1) Given various combinations of open sentences involving multiplication of reciprocals, the pupil demonstrates his knowledge that multiplication of reciprocals yields 1, by replacing the open frames with the correct numeral.

2) Given a multiplication example involving two fractional numbers, the pupil uses cancelling to reduce the work involved in the actual multiplication algorithm to solve the product.

Division of Fractions I.
1) Given an open multiplication sentence where the first factor is missing and the second factor and product are given, the pupil supplies the missing factor by direct observation.

2) Given the related open division sentence, the pupil supplies the missing quotient by recognizing the relationship between multiplication and division.

Division of Fractions II.
1) Given a division example where a fractional number is divided by a whole number, the pupil computes the quotient and writes the answer in simplest terms.

2) Given a division example involving two proper fractions, the pupil computes the quotient and writes the answer in simplest terms.

Problem Solving V.
1) Given a multiplication or division statement problem involving fractional numbers, the pupil writes the name of the appropriate operation used to solve the problem.

2) Given a multiplication or division statement problem involving fractional numbers, the pupil uses the appropriate operation to solve the problem.

Problem Solving VI.
1) Given an addition, subtraction, multiplication, or division statement problem involving fractional numbers, the pupil writes the name of the appropriate operation used to solve the problem.

ENRICHMENT WORKSHEET 2

Probability

Name ______________________________________

FOLD BACK HERE

3) Jack had a pair of dice. He wanted to see how many times he could roll the number seven. He made a table that looked like this:

	1st die	2nd die	Combination
1st try	4	1	(4+1)= 5
2nd	6	6	(6+6)= 12
3rd	5	2	(5+2)= 7
4th	1	1	(1+1)= 2
5th	4	5	(4+5)= 9
6th	5	4	(5+4)= 9

After rolling the dice 6 times, the combination of 7 came up only once. Jack began to wonder how many different outcomes could he have when he rolled the dice. Jack thought of a way to count them. He made a list that looked like this:

(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)
(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)

The first number shows the number appearing on the top of the first die. The second number of each pair shows the number appearing on top of the second die.

How many different outcomes are there? __________ 36

Circle the six outcomes that Jack recorded on his first six throws.

(4,1) (6,6)
(5,2) (1,1)
(4,5) (5,4)
FOLD BACK

UNFOLD & CHECK

4) Then Jack made a drawing using dots to represent the outcomes. His drawing looked like this:

```
        6 . . . . . x
        5 . . . x . .
(2nd    4 . . . . x .
 die)   3 . . . . . .
        2 . . . . x .
        1 x . . x . .
          1 2 3 4 5 6
          (1st die)
```

The numbers going across show all of the possible numbers that could appear on the first die. The numbers written up and down show all the possible numbers that could appear on the second die.

The x's on his drawing are to show the outcomes he recorded on his 6 throws. To record an outcome of (5,4), Geometry/Measurement IV.

1) Given the dimensions of a rectangular prism, the pupil computes the volume.

2) Given the dimensions of a rectangular prism, the pupil computes the surface area.

Cartesian Plane

1) Given the coordinates of a point, the pupil plots the point on the cartesian plane.

2) Given the point on the cartesian plane, the pupil writes the coordinates of the point.

Negative Numbers

1) Given a picture of a number line, the pupil writes the correct negative integer to associate with the corresponding point.

2) Given two integers, the pupil compares their value and replaces the open frame with the correct sign, < or > .

Problem Solving VII

1) Given a problem with too little information, the pupil is able to determine what added information is needed.

2) Given a problem with extraneous information, the pupil is able to ignore the added information and proceed to work the problem.

ENRICHMENT WORKSHEET 3

Probability

Name ______________________________

FOLD BACK HERE

Jack went across 5 and up 4.
To record an outcome of (3,4), how many spaces would Jack go across? ________ How many spaces would he go up? ________

UNFOLD & CHECK

3 4

FOLD BACK

5) Jack threw the dice 6 more times. Here are his results:

	1st die	2nd die	Combination
1st try	4	4	8
2nd	2	6	8
3rd	1	6	7
4th	3	2	5
5th	3	6	9
6th	2	4	6

What number appeared on the top of the first die on his 3rd throw? ________

What number appeared on the top of the second die on his sixth throw? ________

Of the six times Jack threw the dice, how many times did he have a total of 7 showing on the dice? ________

UNFOLD & CHECK

1

4

1
(once)

FOLD BACK

6) Jack threw the dice 6 more times. This time he used his picture graph, rather than a table, to record his outcomes.

(2nd die)						
6	•	•	•	•	•	•
5	•	•	•	•	x	•
4	•	x	•	•	•	x
3	•	•	•	x	x	•
2	x	•	•	•	•	•
1	•	•	•	•	•	•
	1	2	3	4	5	6

(1st die)

Can you list his 6 results?
(Remember to go across first, and then up.)

________ ________

________ ________

________ ________

UNFOLD & CHECK

in any order

(1,2) (5,3)

(2,4) (5,5)

(4,3) (6,4)

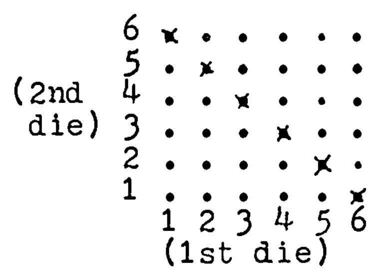
LESSON PLAN AND
TEACHER-CONSTRUCTED MATERIALS
FOR LEVEL 31: ADDITION OF FRACTIONS II

ENRICHMENT WORKSHEET 4

Probability

Name ______________________________ FOLD BACK HERE

7) Jack thought he could use his picture graph to tell how many different ways he could roll 7 with the two dice. He placed an x on his graph to show all the outcomes that would give a combination of 7.



How many ways of rolling 7 are there? ________

6

Can you list them using the short-cut Jack did in making his first list?

________ ________

________ ________

________ ________

(in any order)

(6,1) (3,4)

(5,2) (2,5)

(4,3) (1,6)

UNFOLD & CHECK

FOLD BACK

8) Jack thought if each of the 36 possible outcomes appeared once in 36 throws, he could estimate what the probability or chance of getting a 7 would be. The chance of rolling a 7 is 6 out of 36. Jack used a fraction to show this. $\frac{6}{36}$ The numerator tells how many different ways there are to roll 7. The denominator tells how many different outcomes there are in all.

Can you think of another fraction that names the same value as $\frac{6}{36}$? (hint: Reduce to lowest terms.) ________

$\frac{1}{6}$

Thinking about the fraction in this new way, can you tell how many times you should expect to have a combination of 7 in six throws?

____

1

UNFOLD & CHECK

FOLD BACK

True or false? The probability of getting a 7 when two dice are thrown is $\frac{1}{6}$, or 1 out of 6 tries.

____

true

UNFOLD & CHECK

ENRICHMENT WORKSHEET 5

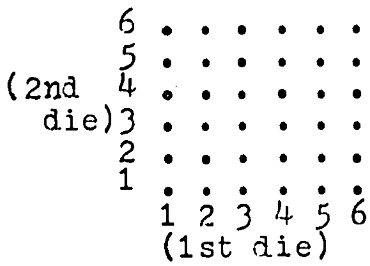
Probability

Name ______________________________

FOLD BACK HERE

9) Use Jack's graph to see how many different ways you can have to roll a 5.

Make a list of them here.



______

______

______

______

UNFOLD & CHECK

(in any order

(1,4)

(2,3)

(3,2)

(4,1)

FOLD BACK

10) Since there are 4 possible ways out of 36, what fraction can you write to show the probability of getting a 5? ______

Reduce the fraction to lowest terms. ______

How many times should you expect to get a 5 showing on the two dice after 9 throws? ______

UNFOLD & CHECK

$\frac{4}{36}$

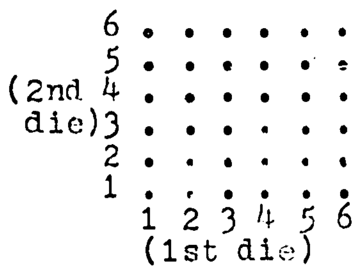
$\frac{1}{9}$

1

FOLD BACK

11) Use Jack's picture graph to tell the following:

What is the probability of getting:



a) a roll of 4 with 2 dice? ______

b) a roll of 2 with 2 dice? ______

c) a roll of 10 with 2 dice? ______

d) a roll of 6 with 2 dice? ______

e) a roll of 7 with 2 dice? ______

f) a roll of 1 with 2 dice? ______

Which number has the greatest probability of turning up, since it has the most possible combinations of occurring? ______

UNFOLD & CHECK

$\frac{3}{36}$ or $\frac{1}{12}$

$\frac{1}{36}$

$\frac{3}{36}$ or $\frac{1}{12}$

$\frac{5}{36}$

$\frac{6}{36}$ or $\frac{1}{6}$

$\frac{0}{36}$ or 0 (It will never happen)

7

REMEMBER: A probability does not tell how many times something will occur for certain. It simply tells us how many times we can expect a thing to happen.

FOLLOW-UP EXERCISES

Probability

Name ______________________________________

FOLD BACK HERE

1) See if you can use this graph to tell the probability of rolling an even number and getting a head when a coin is tossed and a die is cast.

___________ UNFOLD & CHECK

```
6 . .
5 . .
4 . .
3 . .
2 . .
1 . .
  H T
```

$\frac{3}{12}$ or $\frac{1}{4}$

FOLD BACK

2) Draw a graph to show all the possible outcomes you could get in tossing 2 coins.

UNFOLD & CHECK

```
T . .
H . .
  T H
```

FOLD BACK

3) What is the probability of getting a 7 or not getting a 7 when two dice are thrown?

___________ UNFOLD & CHECK

$\frac{36}{36}$ or 1 (a sure thing)

FOLD BACK

4) Take two dice and roll them 36 times. Make a record of your results.

1) _____	10) _____	19) _____	28) _____
2) _____	11) _____	20) _____	29) _____
3) _____	12) _____	21) _____	30) _____
4) _____	13) _____	22) _____	31) _____
5) _____	14) _____	23) _____	32) _____
6) _____	15) _____	24) _____	33) _____
7) _____	16) _____	25) _____	34) _____
8) _____	17) _____	26) _____	35) _____
9) _____	18) _____	27) _____	36) _____

How many times did you roll: 1 _____, 2 _____, 3 _____

4 _____, 5 _____, 6 _____, 7 _____, 8 _____, 9 _____

10 _____, 11 _____, 12 _____? How do your results compare with the probabilities? Were the probabilities good estimates?

ENRICHMENT RECALL SHEET

Probability

Name____________________________________

Date ____________________________________

1) List the three ways you could obtain a combination of 4 when 2 dice are thrown.

_______ _______ _______

2) How many different outcomes are there in throwing 2 coins?

____

3) What is the probability of getting a 2 when two dice are thrown?

____

4) What is the probability of getting an 11 when two dice are thrown?

____

5) What is the probability of getting a tail when one coin is tossed?

____

6) What is the probability of getting two heads when two coins are tossed?

____

LESSON PLAN AND
TEACHER-CONSTRUCTED MATERIALS
FOR LEVEL 45: NEGATIVE NUMBERS

NEGATIVE NUMBERS

A. Behavioral Objectives

1. Given a picture of a number line, the pupil writes the correct negative integer to associate with the corresponding point.

2. Given two integers, the pupil compares their values and replaces the open frame with the correct sign < or > .

B. Procedures

1. Period One (Whole Class)

a. Motivational Technique

Relate this fairy-tale story: One day the whole numbers lined up to go for a walk. (Draw a number line starting with 1 and continuing to 6 or so: ← 1 2 3 4 5 6 →). They marched along until they came to a very round and deep lake. (Add 0 to the number line: ← 0 1 2 3 4 5 6 →). Guess what they saw as they gazed into the lake? (Hear various responses). No, they didn't see any fish. No, they didn't see any frogs. As they gazed into the lake, they saw reflections of themselves. (Add "reflections" on left side of zero, leaving off the negative signs for now: ← 5 4 3 2 1 0 1 2 3 4 5 →). Now the numbers had never seen themselves before. But as number 1 looked at the "reflections" he saw that something was different. He saw that their "reflections" were really opposites of what the numbers actually were. Number 2 was not so sure. At first he couldn't see any difference. But all of the numbers looked at their "reflections" and they were positive (Add + signs to numbers on the right of zero: ← 6 5 4 3 2 1 0 +1 +2 +3 +4 +5 +6 →) that the "reflections" were different from what they were. Since they were so positive, we'll call these the "positive numbers". (Write the words "positive numbers" under the positive integers). This sign (+) shows that the numbers are positive. Now we said that the "reflections" were the opposites of the positive numbers. What sign do you think we will write to show that these numbers (Point to the negative integers on the left of zero) are the opposites of these positive numbers? That's right, we'll use a small - sign. But we'll call it "negative" instead of "minus". (Add - signs to negative integers and write the words "negative numbers" under those numerals: ← -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5 +6 →).

negative numbers   positive numbers

(Point to a +3). This number is a positive three. This number (Point to a -3) is a negative three. Those two numbers are opposites. What is the opposite of a positive 2? Can someone point to it on the number line? Continue to give pupils practice in using the terms positive and negative while associating the correct numeral with the corresponding point on the number line.

b. Discuss the idea of having values less than zero. Include discussion of how negative numbers would be used on a thermometer; for telling the amount of time before and after a rocket blast-off; for keeping score when you "go in the hole"; to talk about places above or below sea level. Encourage students to relate real-life situations that could make use of negative numbers.

c. Teach second behavioral objective.
Suggested Activities:
Use the number line to discuss the values of various integers in relationship to other integers. Be sure to relate this activity to the practical experiences discussed earlier. Review the meaning of the signs $<$ and $>$. Use the criterion of one integer is greater than another integer if it is to the right of that integer on the number line to provide practice in replacing open frames with $<$ or $>$ in exercises such as -4 ____ -5.

2. Period Two (Whole Class)

a. Administer performance sheet.

b. Pupil scoring (optional).

c. Teach review work to be used in maintenance activities. (Suggested topics: decimal place value to thousandths, and procedures for finding the least common denominator).

d. Have pupils survey selected maintenance activities from textbooks. (Example: Page 257 in Addison Wesley and selected exercises from page 193 of the Ginn series may be used for this week's maintenance work). Discuss directions. Have pupils begin work in their notebooks if time permits.

Planning for Differentiated Instruction

Note: Pupils who have NOT earned either or both goals on their performance sheet constitute Group 1.
Pupils who have passed BOTH objectives constitute Group 2.

3. Period Three

   a. Seat pupils according to groups.

   b. Group 1 works on maintenance activities.

   c. Teach "Addition and Subtraction of Negative Numbers" to Group 2 (25 min.).
      * Behavioral Objectives:
        1) Given an addition example involving positive and/or negative integers, the pupil computes the sum.

        2) Given a subtraction example involving positive and/or negative integers, the pupil computes the difference.

      Procedures:

      Materials Needed: Strips of paper in the shape of letters marked as bills or checks to various amounts.

      1) Motivational Technique:
         How many of you like to get mail from the postman? Do your mothers and dads always like to get mail? (Elicit remarks about how nice it is to get letters from friends or even checks, but that bills usually aren't welcome).

      2) Teach behavioral objectives.
         Suggested activities:
         We're going to play a game today with "Postman Stories" (Davis, 1964). To do this we'll have to invent an arithmetic for numbers with signs. (Explain that a +3 will mean the postman has brought a check for $3 and a -2 represents a bill for $2). Suppose the postman brings you a check for $4 and a bill for $2. Are you richer or poorer? By how much? Can anyone think of a number sentence that explains this? (Write the sentence: +4 + -2 = +2 after the children have correctly responded).
         Have volunteers come up to the front of the group and draw "postal cards" from the mail bag. Another child may then write the number

sentence to correspond to that "number story". Two bills drawn for \$3 and \$4 would be represented by the number sentence $-3 + -4 = -7$, which is interpreted as owing \$7 or being \$7 poorer. Practice with the exercise until most children have the general idea.
Most postmen are very good workers and they always bring the mail to the right place. But suppose our imaginary postman often gets things mixed up. For example, if the postman comes on Monday and says, "The check I left you last week was really for the lady next door," and he takes back the check for \$13, does his visit make you richer or poorer? Suppose he gives you a check for \$3 and takes back a check for \$5. Would you be richer or poorer? By how much? Can anyone think of a number sentence that will represent that story?
(Write the sentence in words and symbols for the children to compare:

The postman brought	
you a check	+
for \$3	+3
but also	
took away	+3 −
another check	+3 − +
for \$5.	+3 − +5
Because of his visit,	
you were	
\$2 poorer.	+3 − +5 = −2

Continue with such activities until all possibilities are discovered: adding two positives, subtracting two positives, adding two negatives, subtracting two negatives, adding a negative and a positive, subtracting a negative from a positive, subtracting a positive from a negative).

3) Have pupils in Group 2 work on maintenance activities.

4) Review the ideas presented in Negative Numbers with Group 1 (10 min.).
Suggested Activities:
a) Draw a number line on the board. Have members of the group take turns in supplying numerals to represent corresponding points.

b) Use the number line to determine whether one point is less than or greater than another by comparing the relative position of each point.

4. Period Four

   a. Distribute sets of instructional worksheets to Group 1. Pupils begin work independently.

   b. Review material covered in period three with Group 2. (Extension of the previous activities can be used or additional activities can be taught according to the discretion of the teacher).
   Distribute enrichment worksheets to Group 2; pupils begin work.

   c. Spend remaining time spot-checking all groups, giving individual help where needed.

5. Period Five

   a. Groups 1 and 2 continue with previous day's work, or if finished continue work in maintenance activities.

   b. Spot-check both groups. Give individual help as needed.

   c. Before end of period, check the enrichment recall sheet with Group 1, which will serve as an evaluation instrument.

   d. If pupils are finished with assignments before the end of the period, allow them to play arithmetic games or solve "star problems" from the problem box.

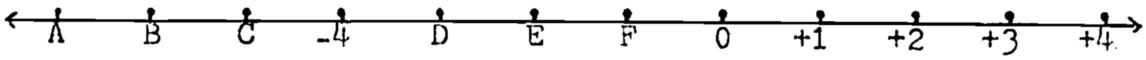
PERFORMANCE SHEET

Negative Numbers

Name ______________________________

Date ______________________

Directions: What numerals would you associate with these points?



1. A = ________ 1. ☐
2. B = ________ 2. ☐
3. C = ________ 3. ☐
4. D = ________ 4. ☐
5. E = ________ 5. ☐
6. F = ________ 6. ☐

Directions: Replace the ___ with the correct sign, < or > .

1. -5 _____ -2 1. ☐
2. +6 _____ -8 2. ☐
3. -2 _____ +1 3. ☐
4. 0 _____ +8 4. ☐
5. -4 _____ -6 5. ☐
6. +3 _____ +2 6. ☐

TOTALS		
GOALS	5	5

ENRICHMENT WORKSHEET

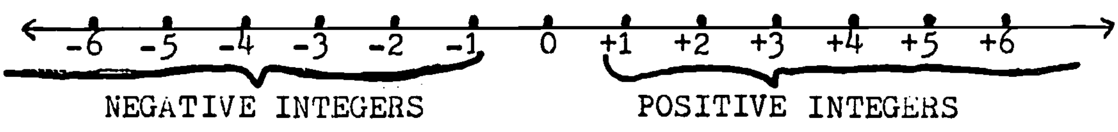
Negative Numbers

Name ______________________________

Date ____________________

FOLD BACK HERE

It is interesting to think about numbers for the points opposite the whole numbers on the number line. Numerals for these numbers are placed to the left of 0 and are written with small - signs before them. These "new" numbers, together with the whole numbers, make up a set of numbers called the integers. The new numbers are called negative integers. The whole numbers other than 0 are called positive integers. Study this picture.



1) Suppose the temperature falls 5° below zero. We can use negative integers to show this. We can represent this temperature by writing -5°.
What would 10° below zero be written as? ____________

-10°

If the weather bureau records the temperature as $+16^\circ$, does this mean that it is above or below zero? ____________

above

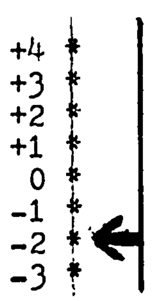
The weather report reads:
TEMPERATURE: (9 A.M.) -7°.
Tell in words what this means. ____________

7 degrees below 0

UNFOLD & CHECK

FOLD BACK

2) Here is a picture of a meter in an elevator. The ground floor is represented at 0. The floors above ground are represented by positive integers. Those below ground are represented by the negative integers.



Did the elevator stop above or below ground? ____________

below

On what numeral would the arrow point if the elevator were on the first floor? ____________

+1

on the first floor below ground level? ____________

-1

UNFOLD & CHECK

ENRICHMENT WORKSHEET 2

Negative Numbers

Name ______________________________

FOLD BACK HERE

3) John was watching a rocket launching on television. He wanted to keep a record of all that happened. He wrote in his notebook:

-6 hrs.	: Astronauts enter spaceship
-1 hr.	: Astronauts make final check
-32 min.	: Final adjustment of flight plan made by computer
-10 min.	: President wishes astronauts a safe trip on telephone
-1 min.	: Final countdown begins
-10 sec.	: Rockets are fired
0	: Liftoff
+59 sec.	: First phase of rocket separates
+4 min.	: Second phase of rocket separates
+2 hrs.	: Astronauts complete first orbit of earth

What set of numbers did John use to show the time before the launching?

____

negative integers

What set of numbers did John use to show the time after the launching?

____

positive integers

If the astronauts ate breakfast 8 hours before the launching, what numeral would John use to record this time in his notebook?

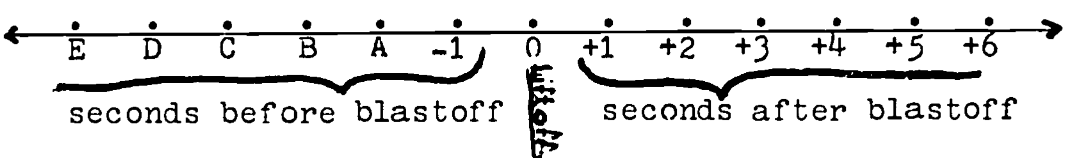
____

-8

UNFOLD & CHECK

FOLD BACK

4) Russ drew this picture graph.



What numerals would you use to replace the letters?

A ______ B ______

C ______ D ______

E ______

-2 -3

-4 -5

-6

UNFOLD & CHECK

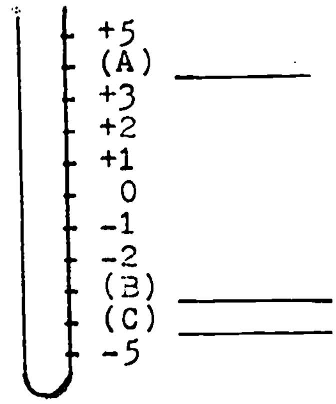
ENRICHMENT WORKSHEET 3

Negative Numbers

Name ____

FOLD BACK HERE

5) Kriss made this picture of a thermometer. Can you fill in the blanks with the correct numerals?



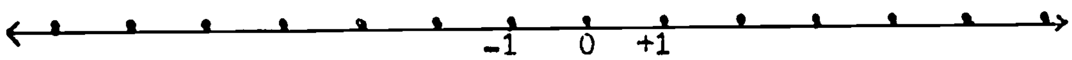
UNFOLD & CHECK

(A) +4

(B) -3
(C) -4

FOLD BACK

6) Fill in this number line using positive and negative integers.



Use your picture of the number line to answer these questions:

Which is more: +7 or +5? __________

Is +7 to the right or left of +5 on the number line? ______________

Which is more: +6 or 0? __________

Is +6 to the right or left of 0 on the number line? ______________

UNFOLD & CHECK

+7

right

+6

right

FOLD BACK

Is 0 to the right or left of -2? ____________________

Which is more: 0 or -2? __________

Is -5 to the right or left of -7? ____________________

Which is more: -5 or -7? __________

Complete:
One integer is greater than another integer if it is to the ______________ of that integer on the number line.

UNFOLD & CHECK

right

0

right

-7

right

ENRICHMENT WORKSHEET 4

Negative Numbers

Name ______________________________________

FOLD BACK HERE

+6 is greater than +4 because +6 is on the right of +4 on the number line.

-7 is greater than -10 because -7 is on the ____________ of -10 on the number line.

UNFOLD & CHECK

right

FOLD BACK

7) Circle the one that has the greater value. Remember the picture of the number line.

a) +4, +6	g) -3, +2
b) 0, +2	h) -4, 0
c) -5, +3	i) 0, -2
d) +5, -1	j) +8, -9
e) +6, -10	k) +6, -1
f) -4, 0	l) +2, -4

UNFOLD & CHECK

a) +6	g) +2
b) +2	h) +4
c) +3	i) 0
d) +5	j) +8
e) +6	k) +6
f) 0	l) +2

FOLD BACK

Is this statement true? All positive integers are greater than negative integers. ____________

yes

+5 is greater than +2. Is -5 greater than -2? ______

no

Why not? ______________________________________

____

UNFOLD & CHECK

-5 is not to the right of -2 on the number line.

FOLD BACK

8) Circle the one that has the greater value. Remember the picture of the number line.

a) -5, -2	g) -12, -1
b) -8, 0	h) -2, -3
c) -4, -3	i) -4, -2
d) -6, -9	j) -1, -6
e) -5, -1	k) -8, -2
f) -2, -3	l) -5, -3

UNFOLD & CHECK

a) -2	g) -1
b) 0	h) -4
c) -3	i) -2
d) -6	j) -1
e) -1	k) -2
f) -2	l) -3

ENRICHMENT WORKSHEET 5

Negative Numbers

Name ______________________________

FOLD BACK HERE

9) Replace the blanks with the correct sign, < or > . (Remember, the wide part of the sign always faces the larger numeral.)

a) +4 ________ 0

b) +2 ________ -4

c) -5 ________ -8

d) -2 ________ -1

e) +5 ________ +2

f) -7 ________ +7

g) +8 ________ -9

h) -2 ________ 0

i) -6 ________ -8

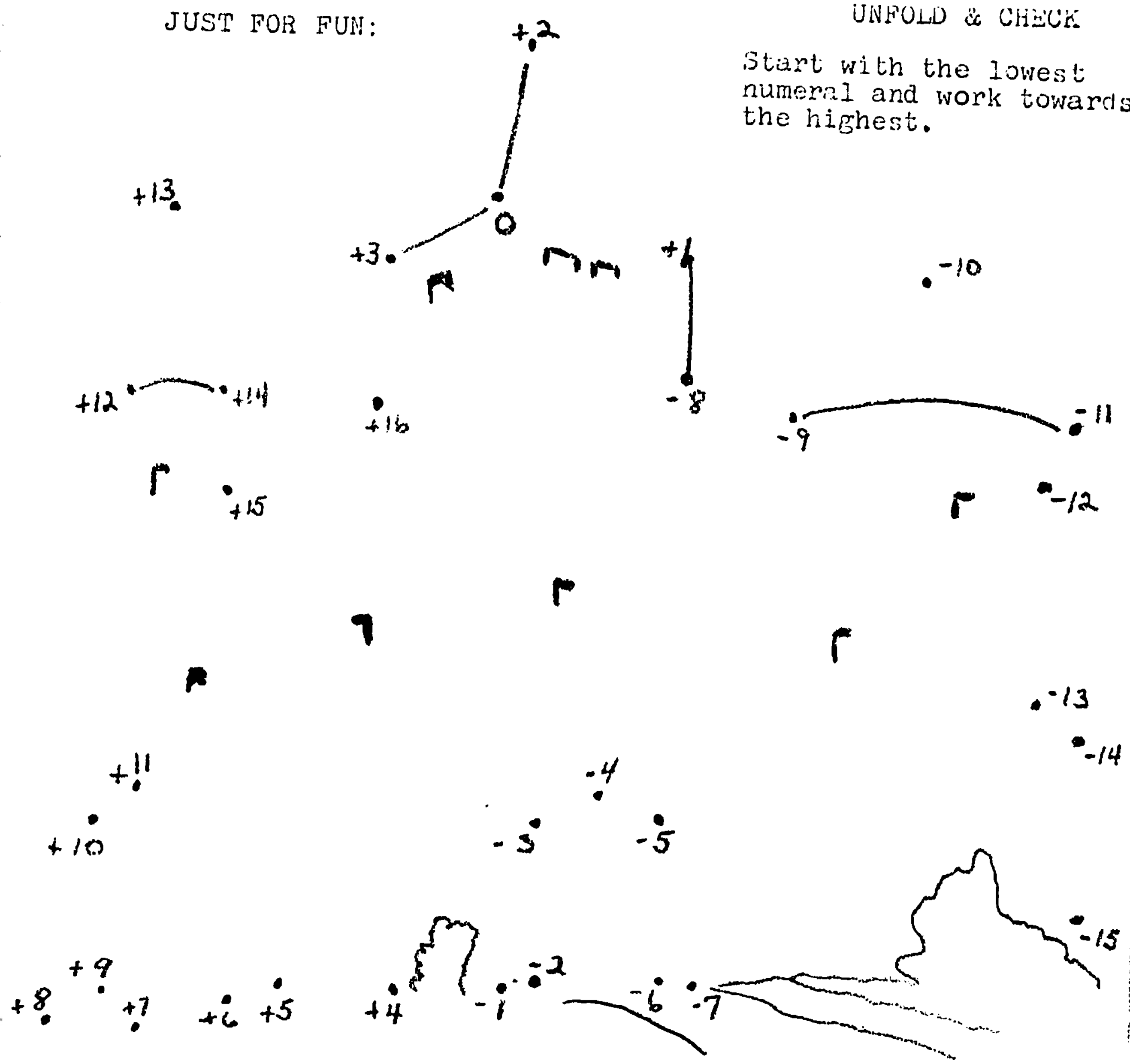
j) -9 ________ -5

a)	f)
b)	g)
c)	h)
d)	i)
e)	j)

UNFOLD & CHECK

JUST FOR FUN:

Start with the lowest numeral and work towards the highest.



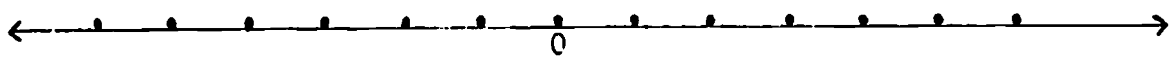
ENRICHMENT RECALL SHEET

Negative Numbers

Name ______________________________________

Date ____________________

1) On this number line, label all of the integers from +6 to -6.



2) Replace the blanks with the correct sign, < (less than), or > (greater than).

a) +4 _____ -6

b) -6 _____ -2

c) -5 _____ +3

d) 0 _____ -4

e) -8 _____ -1

f) +3 _____ +2

ENRICHMENT WORKSHEET

Negative Numbers (+ & -)

Name ______________________________

Date ______________________________

FOLD BACK HERE

1) Write a number sentence for the story.
The postman brings you a check for $7 and a bill for $5.

____

Are you richer or poorer? ______________

By how much? ______________

UNFOLD & CHECK

+7 + -5 = ?

richer

+2 or $2

FOLD BACK

2) Write in words what this sentence means: -5 + -2 = -7

____

____

____

UNFOLD & CHECK

The postman brought a bill for $5 and a bill for $2, which makes you $7 poorer

FOLD BACK

3) Can you make up a postman story for each problem? Use your story to figure out an answer for each problem. Write your answer on the line.

a) +6 + +2 = ______

b) +8 + -1 = ______

c) -5 + -4 = ______

d) +2 + -5 = ______

e) -4 + +9 = ______

f) -6 + +2 = ______

g) -4 + 0 = ______

UNFOLD & CHECK

a) +8

b) +7

c) -9

d) -3

e) +5

f) -4

g) -4

ENRICHMENT WORKSHEET 2

Negative Numbers (+ & -)

Name ______________________________

FOLD BACK HERE

4) Write a number sentence for this story.
The postman brings you a check for $3 and takes back a bill for $3.

____

Are you richer or poorer? ____________

By how much? ____________

UNFOLD & CHECK

$+3 - -3 = ?$

richer

$+6$ or $6

FOLD BACK

5) Write a number sentence for this story.
The postman brings a bill for $5 and takes back a bill for $7.

____

Are you richer or poorer? ____________

By how much? ____________

UNFOLD & CHECK

$-5 - -7 = ?$

richer

$+2$ or $2

FOLD BACK

6) Write a story for what this sentence means:
$+2 - -3 = +5$.

____

____

____

UNFOLD & CHECK

The postman brings a check for $2 and takes back a bill for $3, which makes you $5 richer.

FOLD BACK

7) Write a story for what this sentence means:
$-5 - +8 = -13$.

____

____

____

UNFOLD & CHECK

The postman brings a bill for $5 and takes away a check for $8, which makes you $13 poorer.

FOLD BACK

8) Write a number sentence for this story.
The postman brings a check for $5 and a bill for $1, and then takes back a check for $3.

____

How much richer or poorer are you? ____________

UNFOLD & CHECK

$+5 + -1 - +3 = ?$

$1 richer

ENRICHMENT WORKSHEET 3

Negative Numbers (+ & -)

Name ____

9) Can you make up a postman story for each problem? Use your story to figure out an answer for each problem. Write your answer on the line.

a) +4 - +2 = ________

b) +6 - -3 = ________

c) +2 - +7 = ________

d) -4 - +6 = ________

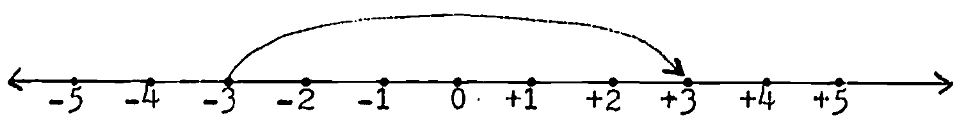
e) -3 - -2 = ________

f) -6 - +1 = ________

g) 0 - +3 = ________

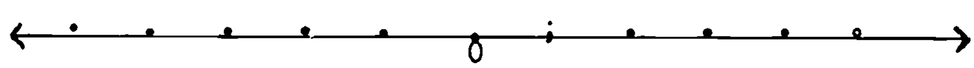
UNFOLD & CHECK

10) Tom made this picture



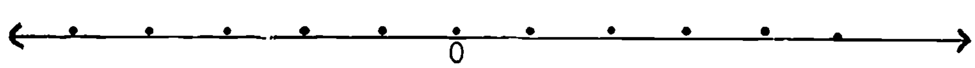
to show that the opposite of -3 is +3. We can use these symbols to say the same thing. $^{o}(-3) = +3$

See if you can make a picture to show $^{o}(+1) = -1$.



UNFOLD & CHECK

11) See if you can make a picture to show $^{o}(-5) = +5$.



UNFOLD & CHECK

FOLD BACK HERE

a) +2

b) +9

c) -5

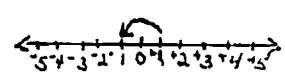
d) -10

e) -1

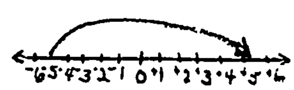
f) -7

g) -3

FOLD BACK



FOLD BACK



ENRICHMENT WORKSHEET 4

Negative Numbers (+ & -)

Name ______________________________

12) What is +4 + -4 = ______?

-4 + +4 = ______?

UNFOLD & CHECK

FOLD BACK HERE

0

0

FOLD BACK

13) What values make these sentences true?

a) +8 + ______ = 0   c) +6 + ______ = 0

b) -8 + ______ = 0   d) -6 + ______ = 0

UNFOLD & CHECK

a) -8   c) -6

b) +8   d) +6

FOLD BACK

14) Fill in the missing numerals.

a) +9 - ______ = 0

b) -9 - ______ = 0

c) +9 + ______ = 0

d) -9 + ______ = 0

e) 0 - ______ = +9

f) 0 - ______ = -9

g) +9 - ______ = +9

h) -9 + ______ = -9

i) 0 + ______ = +9

UNFOLD & CHECK

a) +9

b) -9

c) -9

d) +9

e) -9

f) +9

g) 0

h) 0

i) +9

FOLD BACK

*** Problems

Name the next 4 numbers in each sequence.

a) -4, -2, 0, +2, ______, ______, ______, ______

b) +3, +2, 0, -3, ______, ______, ______, ______

c) +5, -1, -7, -13, ______, ______, ______, ______

UNFOLD & CHECK

+4,+6,+8,+10

-7,-12,-18,-25

-19,-25,-31,-37

CONCLUSION

The need for teacher-developed curriculum innovations is highly recognizable in the reconstruction of educational practices. Many new proposals conceived and developed by project staffs appear to lose much of their innovative character when an attempt is made to implement them. The teacher is almost always the key to the success or failure of an innovative program. This fact suggests that teachers are in a predominate position to bring about necessary changes. Little is done, however, in educating teachers to this responsibility, and it is no wonder that improvements in the educational scene are slow or unproductive.

It has long been the existing practice of educational reform to place the role of reconstructing educational practices in the hands of "specialists" who will in turn lead teachers to improving learning situations. It is the purpose of this paper, to suggest an alternate approach to meeting the need for new patterns in educational practice. A teacher with the proper training should be given the opportunity to develop and implement a curriculum of her own design. This endeavor places the teacher in greater active involvement in effecting change by shifting her role from follower to leader. In accepting a role of innovator, a teacher is no longer simply exposed to new ideas; she is engaged in internalizing them in a fuller meaning since she is involved in implementing her own ideas.

This paper presents a case in point. The teacher's approach to constructing a mathematics curriculum presented here, is a subjective one, and is not intended to be considered as the only approach. It is hoped that the method of approaching the task of developing and implementing a curriculum as outlined in this paper will serve as a guide or encouragement to other teachers to undertake a similar endeavor. Since the development and implementation of this fifth grade program is still in progress, to be completed in the course of the school year, no significant evaluation can be made at this time. However, certain worth-while assurances can already be recognized through this undertaking.

> Operationally schematizing mathematics into fundamental components lends multiple assurances.
> 1) The teacher must have depth of understanding of mathematics to schematize its structure in the first place.
> 2) She must make some assumptions about degrees of importance to be assigned mathematical components for purposes of teaching.
> 3) If the teacher attaches any importance to her own assumptions (arrived at in a classroom context) she will be less bound to the textbook and will be better equipped to meet individual differences.
> 4) Operating deductively and/or inductively, depending on the point of departure, helps to assure that the participant - child or teacher - grasps the significance of intertwining conceptual structures.
> 5) The participant should gain greater understanding of ways in which mathematics is learned.
> 6) The organizing structure of mathematics itself is learned. (Frost & Rowland, pp.342-343).

When the teacher constructs and implements her own curriculum she hopes to provide a better learning situation for her students. If, however, after careful evaluation, no significant difference is noted between instruction in the traditional program and that in the innovative program, some benefit will still be assured. For in undertaking the task of adjusting instruction to meet the needs of different ability groups, the teacher will have increased her knowledge and understanding of the subject, thereby improving her abilities as a teacher.

REFERENCES

Carman, Robert A., and Carman, Marilyn J. "Number Patterns," Arithmetic Teacher, 17:637-639, December, 1970.

Cook, Nancy. "Fraction Bingo," Arithmetic Teacher, 17:237-239, March, 1970.

Davis, Robert B. Discovery In Mathematics, The Madison Project Student Discussion Guide, Palo Alto: Addison-Wesley Publishing Company, 1964.

Deans, Edwina. Elementary School Mathematics: New Directions, Washington: U.S. Government Printing Office, 1963.

DeVault, M. Vere. Improving Mathematics Programs, Trends and Issues in the Elementary School, Columbus, Ohio: Charles E. Merrill Books, Inc., 1961.

Eicholz, Robert E., and O'Daffer, Phares. Elementary School Mathematics: Book 5, Menlo Park, California: Addison-Wesley Publishing Company, 1968.

Eisner, Elliot (ed.). Confronting Curriculum Reform, Boston: Little Brown and Company, 1971.

Frost, Joe L., and Rowland, G. Thomas. Curricula for the Seventies, Early Childhood Through Early Adolescence, Boston: Houghton Mifflin Company, 1969.

Hartley, Rogers Jr. "The Euclidian Algorithm as a Means of Simplifying Fractions," Arithmetic Teacher, 17:657-652, December, 1970.

Ibe, Milagros D. "Better Perception of Geometric Figures through Folding and Cutting," Arithmetic Teacher, 17:583-586, November, 1970.

Kibler, Robert J., et al. Behavioral Objectives and Instruction, Boston: Allyn and Bacon, Inc., 1970.

Kramer, Klaas (ed.). Problems in the Teaching of Elementary School Mathematics, Boston: Allyn and Bacon, Inc., 1970.

Lee, J. Murray. Elementary Education Today and Tomorrow, Boston: Allyn and Bacon, Inc., 1967.

Mager, Robert F. Preparing Instructional Objectives, Palo Alto, California: Fearon Publishers, 1962.

May, Lola J. "What are the Odds?", Grade Teacher, 88: 62-63, March, 1971.

Morgenstern, Ann (ed.). Grouping in the Elementary School, New York: Pitman Publishing Co., 1966.

Sauble, Irene, and Brownell William. Teaching Mathematics We Need, Boston: Ginn and Company, 1966.

Schools Council for the Curriculum and Examinations. Mathematics in Primary Schools, London: Her Majesty's Stationery Office, 1965.

School Mathematics Study Group. Report of A Conference On Elementary School Mathematics Chicago, Illinois, February 13 and 14, 1959.

Shipp, Donald E., and Adams, Sam. Developing Arithmetic Concepts and Skills, Englewood Cliffs, N.J.: Prentice Hall, Inc., 1964.

Suydam, Marilyn N., and Riedesel, C. Alan. Interpretive Study of Research and Development in Elementary School Mathematics, Volume 3; Developmental Projects, U.S.O.E., Final Report Project No. 8-0586, Grant No. OEG-0-9-480586-1352(010), University Park, Pa.: The Pennsylvania State University, 1969.

Vigilante, Nicholas J. (ed.). Mathematics in Elementary Education, Selected Readings, Toronto, Ontario: Macmillan Company, 1969.

Walter Marion. "An Example of Informal Geometry: Mirror Cards", Arithmetic Teacher, 13:448-452, October, 1966.

. . . Manual for A Systems Approach to Improving Mathematics Instruction (SAM), The Board of Public Education, Pittsburgh, Pennsylvania, 1970.